Physics-Based Sound Field Reproduction

9th Conference on Speech Technology and Human-Computer Dialogue
SpeD 2017, July 6-9, 2017, Bucharest, Romania

Rudolf Rabenstein

Rudolf.Rabenstein@fau.de

Chair for Multimedia Communications and Signal Processing
Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Germany

July 6, 2017

Contents

- Overview on methods for sound field reproduction
- Focus on physically correct reproduction of sound fields
- Discussion focussed on to two families of reproduction methods:
  - Wave Field Synthesis
  - Ambisonics
- Only few mathematical equations,
  instead graphical representations of sound fields

3 Natural Sound Field

Monofrequent sound field of a point source

human hearing:

localization by

- interraural time differences
- interaural intensitiy differences

sound recording with microphones:

localization by time differences
(time difference of arrival, TDoA)

Stereophonic reproduction of a point source (two channel stereo)

Superposition
of the two sound fields
of each loudspeaker creates
interference patterns
which are locally simular
to the sound field
of a virtual point source.

Stereophonic reproduction for multiple listener positions

Listeners in the center
and on the right
perceive a sound source in
between the speakers.

Listener on the left
resides a the null
of the interference pattern.

Center position:

sweet spot

Stereophonic reproduction for multiple listener positions
at a different frequency

Listeners on the left
and in the center
perceive a sound source in
between the speakers.

Listener on the right
resides a the null
of the interference pattern.

Stereophonic reproduction of sound fields

two channel stereo	left – right
5.1 surround sound	left – center – right + two rear channels
several cinema formats	

- designed for human perception
  target: listener in the sweet spot
- driven by entertainment requirements

music	orchestra or band setup
cinema	dialogue, music and sound effects

- advantage:
  popular and widespread, extremely large selection of recordings
- disadvantage:
  loudspeaker setup depends on reproduction format
  (stereo, 5.1, home cinema, cinema)

Can we design sound fields from elementary components?

... similar to the design of wave forms from sinusoids?
(Fourier series, Fourier transformation, and alike)

time signal $v(t)$
t time

$$v(t) = \sum_{\nu} V_{\nu} \exp(j\omega_{\nu} t)$$

summation with respect to discrete frequencies ω_{ν}

V_{ν} expansion coefficients

real part of $\exp(j\omega_{\nu} t)$:
sinusoidal signal

sound pressure field $p(\boldsymbol{x}, t)$
$\boldsymbol{x}$ space

$$p(\boldsymbol{x}, t) = \sum_{\nu} P_{\nu} \exp\left(j\omega_0 \left(t + \tfrac{1}{c}\boldsymbol{n}_{\nu}^{\mathrm{T}}\boldsymbol{x}\right)\right)$$

summation with respect to discrete normal directions $\boldsymbol{n}_{\nu}$

P_{ν} expansion coefficients
ω_0 fixed frequency
c speed of sound

real part of $\exp\left(\omega_0 \left(t + \tfrac{1}{c}\boldsymbol{n}_{\nu}^{\mathrm{T}}\boldsymbol{x}\right)\right)$:
monofrequent plane wave

because $ct + \boldsymbol{n}_{\nu}^{\mathrm{T}}\boldsymbol{x} = 0$ is the Hessian normal form of a plane in space plane wave from the top

Can be approximated
by a far away point source.

5 Fourier Acoustics

plane waves from the top left and top right superposition of two plane waves from the top left and top right

Interference pattern
similar to two channel stereo

superposition of 13 plane waves from equidistant directions all around the circle

Approximation of a circular pattern

5 Fourier Acoustics

Sound fields can be

- composed from plane waves
- decomposed into plane waves

in much the same way as signals can be composed from sinusoids.

Sinusoids:	eigenfunctions of linear and time-invariant systems (LTI-system)
plane waves:	eigenfunctions of the acoustic wave equation (a multidimensional LTI-system)

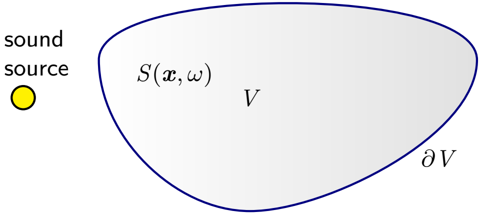
$$c^2\,\Delta p(\boldsymbol{x},t) = \frac{\partial^2}{\partial t^2}\,p(\boldsymbol{x},t)$$

- The eigenfunctions of the acoustic wave equation are determined by the Laplace operator $\Delta = \operatorname{div}\operatorname{grad}$
- The eigenfunctions of the Laplace operator take many different forms depending on the adopted coordinate system (Cartesian, polar, spherical, cylindrical)

There is a rich world of different sound field representations !

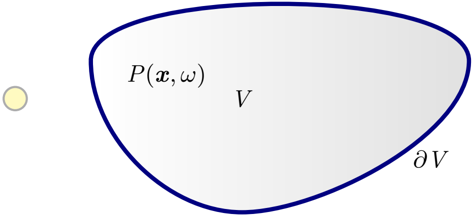
Problem description

Sound field
inside the volume V
with boundary ∂V
caused by an external source



Reproduce
the same sound field by
a distribution of sound sources
on the boundary ∂V
of the volume V

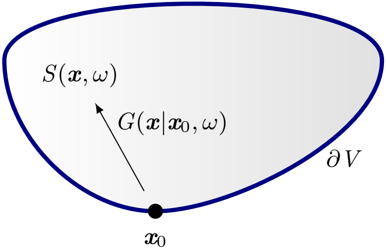
with $P(\boldsymbol{x}, \omega) \approx S(\boldsymbol{x}, \omega)$



Synthesis Equation

$$S(\boldsymbol{x},\omega) = \oint_{\partial V} D(\boldsymbol{x}_0,\omega)\, G(\boldsymbol{x}|\boldsymbol{x}_0,\omega)\, d\boldsymbol{x}_0$$

$\boldsymbol{x} \in V$	arbitrary point inside the volume V
$\boldsymbol{x}_0 \in \partial V$	arbitrary point on the boundary ∂V
$G(\boldsymbol{x}\|\boldsymbol{x}_0,\omega)$	Green's function
$S(\boldsymbol{x},\omega)$	desired sound field
$D(\boldsymbol{x}_0,\omega)$	driving function at $\boldsymbol{x}_0$ (source strength)



Explicit Solution of the synthesis equation

$$S(\boldsymbol{x},\omega) = \oint_{\partial V} D(\boldsymbol{x}_0,\omega)\, G(\boldsymbol{x}|\boldsymbol{x}_0,\omega)\, d\boldsymbol{x}_0$$

- Choose the boundary ∂V as the surface of a sphere with radius R.
- Express the spatial variables in spherical coordinates:

$$\boldsymbol{x} \longrightarrow (r,\alpha,\beta)$$
$$\boldsymbol{x}_0 \longrightarrow (R,\alpha,\beta)$$

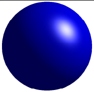
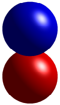
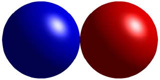
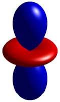
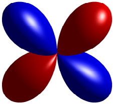
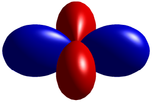
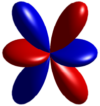
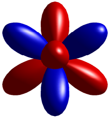
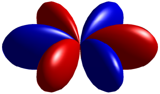
- Expand all functions into spherical harmonics

$$D(R,\alpha,\beta,\omega) = \sum_{m,n} \mathring{D}_n^m(R,\omega)\, Y_n^m(\beta,\alpha)$$

- determine the expansion coefficients $\mathring{D}_n^m(R,\omega)$ of the driving function from the expansion coefficients of
  - the desired sound field and
  - the Green's function.

Foundation for spatial sound reproduction by *Ambisonics*.

Spherical harmonic functions

$n\backslash m$	0	1	2	3
0				
1				
2				
3				

Implicit Solution

Kirchhoff-Helmholtz integral equation

$$P(\boldsymbol{x},\omega) = \oint_{\partial V} \Big(P(\boldsymbol{x}_0,\omega)\, \nabla G(\boldsymbol{x}|\boldsymbol{x}_0,\omega) - \nabla P(\boldsymbol{x}_0,\omega)\; G(\boldsymbol{x}|\boldsymbol{x}_0,\omega) \Big)\, d\boldsymbol{x}_0$$

(derivation: apply Green's second identity to the wave equation)

$P(\boldsymbol{x},\omega)$	sound pressure field
$G(\boldsymbol{x}\|\boldsymbol{x}_0,\omega)$	Green's function of a monopole source (point source)
$\nabla G(\boldsymbol{x}\|\boldsymbol{x}_0,\omega)$	Green's function of a dipole source

Assumption: Only monopoles on the boundary: $\nabla G(\boldsymbol{x}|\boldsymbol{x}_0,\omega) = 0$

$\nabla G(\boldsymbol{x}|\boldsymbol{x}_0,\omega) = 0$

$$P(\boldsymbol{x},\omega) = \oint_{\partial V} \Big(-\nabla P(\boldsymbol{x}_0,\omega) \Big)\, G(\boldsymbol{x}|\boldsymbol{x}_0,\omega)\, d\boldsymbol{x}_0$$

compare synthesis equation

$$S(\boldsymbol{x},\omega) = \oint_{\partial V} D(\boldsymbol{x}_0,\omega)\, G(\boldsymbol{x}|\boldsymbol{x}_0,\omega)\, d\boldsymbol{x}_0$$

Choosing the driving function $D(\boldsymbol{x}_0, \omega)$ as

$$D(\boldsymbol{x}_0, \omega) = -\nabla P(\boldsymbol{x}_0, \omega)$$

synthesizes the desired sound field $P(\boldsymbol{x}, \omega) = S(\boldsymbol{x}, \omega)$.

Further steps:

- Reduce the boundary ∂V of the surface to a manageable area (e.g. a line).
- Sample this area by densely placed individual loudspeakers.
- Provide the driving functions
  $d(\boldsymbol{x}_0, t) = \mathscr{F}^{-1}\{D(\boldsymbol{x}_0, \omega)\}$
  for each loudspeaker.

Foundation for spatial sound reproduction by *Wave Field Synthesis (WFS)*

Sound field reproduction with Ambisonics

Superposition of monopole sound fields from all loudspeakers around the full circle

Sound field reproduction with Wave Field Synthesis

Superposition of
monopole sound fields
from selected loudspeakers

Sound field reproduction with Local Wave Field Synthesis

Reproduction of
a dedicated soundfield
only in a restricted region,
e.g. around the listener.

Potential for
the creation of
multiple independent
listening zones
with the same
loudspeaker setup.

8.0 Wave Field Synthesis

64-channel sound field synthesis system
Signal Theory and Digital Signal Processing (Prof. S. Spors)
University Rostock, Germany

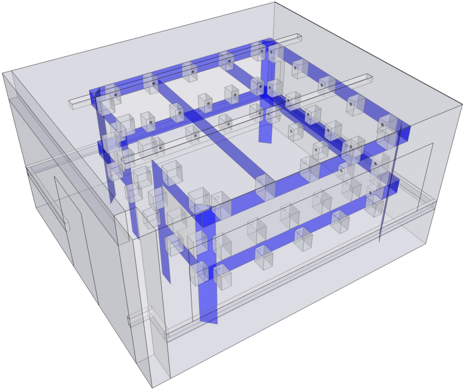


image by S. Spors

8.0 Wave Field Synthesis

128-channel sound field system
Multimedia Communications and Signal Processing
University Erlangen-Nürnberg, Germany

- rectangular array
- 128 speakers
- 128 class-D amplifiers
- 2 64-channel sound cards
- software: Sound Scape Renderer
  http://spatialaudio.net/ssr/

8.0 Wave Field Synthesis

128-channel sound field system
Multimedia Communications and Signal Processing
University Erlangen-Nürnberg, Germany

9 Strenghts and Limitations

Strenghts

- Rigorous derivation from the first principles of acoustics
- Both Wave Field Synthesis and Ambisonics have the capability to produce physically meaningful sound fields
- Hardware components are commercially available
- Operating software is available (including open source software)
- Experience with implemented systems at various institutions throughout Europe
- Further developments under research

Limitations

- Good results require a high number of individual audio channels
- The reproduced sound fields may be impaired by room reflections
- For high frequencies or
  a low number of loudspeakers
  (few loudspeakers per wave length)
  - aliasing effects (WFS and Ambisonics)
  - small sweet spot (Ambisonics)
- For non-circular or non-spherical loudspeaker arrangements:
  trunction effects
  - at the ends or
  - at corners

  of loudspeaker arrays

10 Credits

The sound field visualizations have been created with the

Sound Field Synthesis Toolbox

H. Wierstorf, S. Spors - Sound Field Synthesis Toolbox.
Proceedings of 132nd Convention of the Audio Engineering Society, 2012

Quality & Usability Lab together with Assessment of IP-based Applications
Telekom Innovation Laboratories, TU Berlin

Institut für Nachrichtentechnik
Universität Rostock

`http://github.com/sfstoolbox/sfs`
`http://python.sfstoolbox.org`
`https://doi.org/10.5281/zenodo.345435`

11 Literature

The following publications contain extensive references to the literature on sound field synthesis:

[1] S. Spors, R. Rabenstein, and J. Ahrens, "The theory of wave field synthesis revisited," in *Audio Engineering Society (AES) 124th Convention*, Amsterdam, May 2008, p. Paper Number: 7358.

[2] J. Ahrens, *Analytic Methods of Sound Field Synthesis*, ser. T-Labs Series in Telecommunication Services. Berlin Heidelberg: Springer-Verlag, 2012.

[3] H. Wierstorf and S. Spors, "Sound Field Synthesis Toolbox," in *132nd Convention of the Audio Engineering Society*, 2012.

[4] R. Rabenstein, S. Spors, and J. Ahrens, *Academic Press Library in Signal Processing*. Chennai: Academic Press, 2013, vol. 4, ch. 32, Sound Field Synthesis, pp. 915–979.

[5] S. Spors, H. Wierstorf, A. Raake, F. Melchior, M. Frank, and F. Zotter, "Spatial sound with loudspeakers and its perception: A review of the current state," *Proceedings of the IEEE*, vol. 101, no. 9, pp. 1920–1938, Sept 2013.

[6] J. Ahrens, R. Rabenstein, and S. Spors, "Sound field synthesis for audio presentation," *Acoustics Today*, vol. 10, no. 2, pp. 15–25, 2014, print. [Online]. Available: https://acousticstoday.org/issues/2014AT/Spring2014/

[7] K. Helwani, S. Spors, and H. Buchner, "The synthesis of sound figures," *Multidimensional Systems and Signal Processing*, vol. 25, no. 2, pp. 379–403, 2014. [Online]. Available: http://dx.doi.org/10.1007/s11045-013-0261-4

[8] W. Zhang, P. Samarasinghe, H. Chen, and T. Abhayapala, "Surround by sound: A review of spatial audio recording and reproduction," *Applied Sciences*, vol. 7, no. 6, p. 532, May 2017. [Online]. Available: http://dx.doi.org/10.3390/app7050532

[9] J. Blauert and R. Rabenstein, *Architectural Acoustics Handbook*. Florida USA: J. Ross Publishing, 2017, ch. Room-Related Sound Representation using Loudspeakers.

[10] N. Hahn, F. Winter, and S. Spors, "Local Wave Field Synthesis by Spatial Band-limitation in the Circular/Spherical Harmonics Domain," in *140th Convention of the Audio Engineering Society*, Paris, France, Jun. 2016. [Online]. Available: https://www.int.uni-rostock.de/fileadmin/user_upload/publications/spors/2016/Hahn_etal_AES_local_WFS.pdf